2015-2016 Spring Semester
Material and Energy Balance

Dimensions, Units and Conversion Factors

Assist. Prof. Dr. Murat Alkan

16.02.2016

1st Week

Definitions

- A measured or counted quantity has a numerical **value** and a **unit** (2 meters, 45 seconds, 5 kilograms, etc.)
- A **dimension** is a property that can be measured, such as length, time, mass or temperature, or obtained by manipulating other dimensions.
- Dimensions are specified by giving the value relative to some arbitrary standard called a **unit**.
- The complete specification of a dimension must consist of a <u>number</u> and a <u>unit</u>.
- There are two common systems of units used in engineering calculations: American engineering system (**AES**) and Le System Internationale d'Unites (**SI**)

System of Units

- A system of units has the following components:
  - Base units : mass, length, time, temperature, electric current and light intensity
  - Multiple units : multiples or fractions of base units
  - Derived units : by multiplying and dividing base or multiple units.

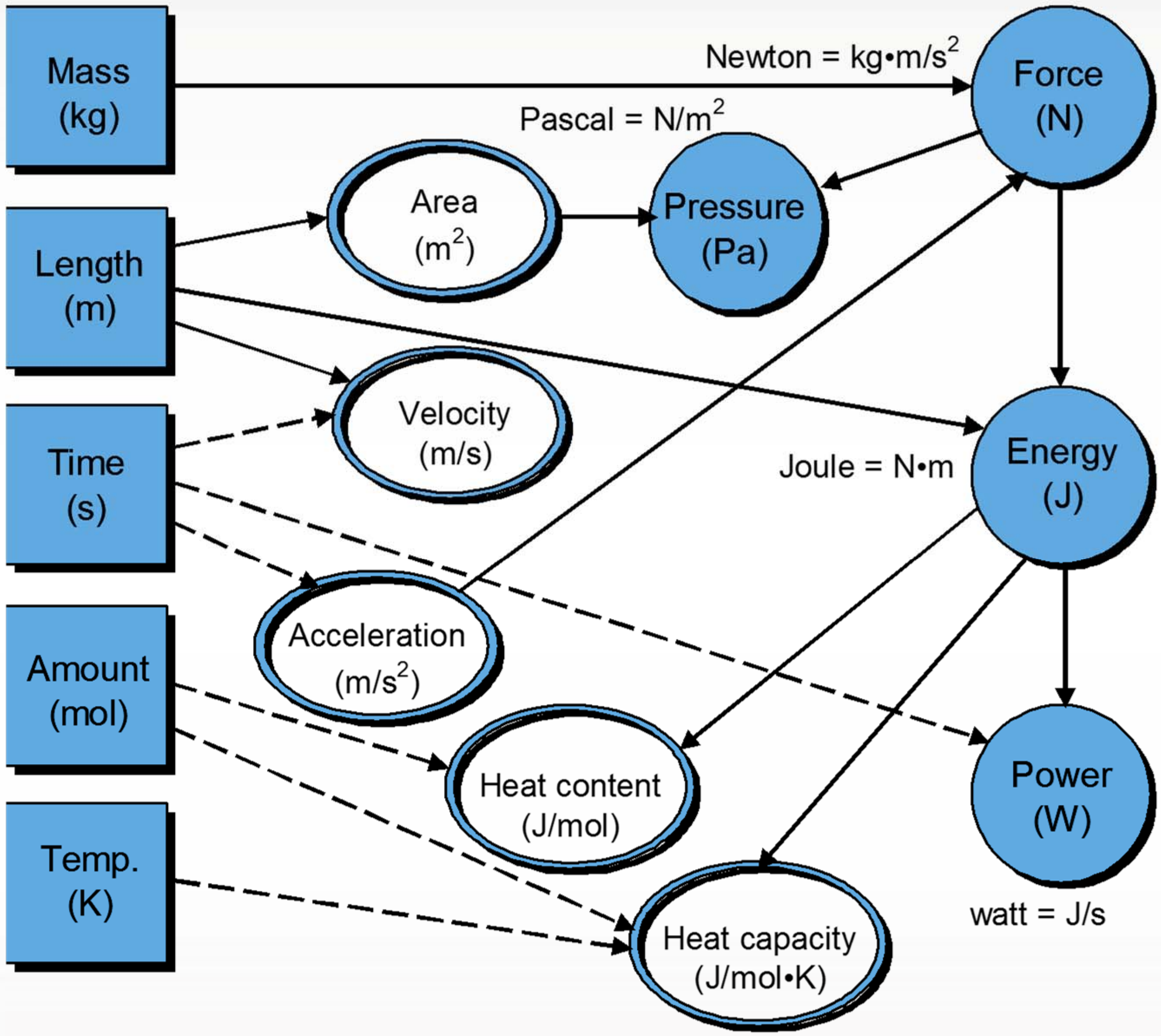
Base Quantity	SI name	SI symbol	AES name	AES symbol
Length	meter	m	foot	ft
Mass	kilogram	kg	pound(mass)	lb_m
Time	second	s	second	s
Electric current	ampere	A		
Temperature (t) Temperature (T)	Celsius Kelvin	°C K	Fahretheit Rankine	°F °R
Amount of substance	gram-mole	g-mol	pound-mole	lb-mol
Luminous (light) intensity	candela	cd		

Derived Units

	Derived SI units		
Force	newton	N	$(kg)(m)(s^{-2}) \rightarrow (J)(m^{-1})$
Energy	joule	J	$(kg)(m^2)(s^{-2})$
Power	watt	W	$(kg)(m^2)(s^{-3}) \rightarrow (J)(s^{-1})$
Density	kilogram per cubic meter		$(kg)(m^{-3})$
Velocity	meter per second		$(m)(s^{-1})$
Acceleration	meter per second squared		$(m)(s^{-2})$
Pressure	newton per square meter, pascal		$(N)(m^{-2})$, Pa
Heat capacity	joule per (kilogram × kelvin)		$(J)(kg^{-1})(K^{-1})$

	Derived units	
Force	pound (force)	lb_f
Energy	British thermal unit, foot pound (force)	Btu, $(ft)(lb_f)$
Power	horsepower	hp
Density	pound (mass) per cubic foot	lb_m/ft^3
Velocity	feet per second	ft/s
Acceleration	feet per second squared	ft/s^2
Pressure	pound (force) per square inch	$lb_f/in.^2$
Heat capacity	Btu per pound (mass) per degree F	$Btu/[(lb_m)(°F)]$
Volume	cubic feet	ft^3

Relationship Between Selected SI Units



Solid lines indicate multiplication, dashed lines division.

- A notable difference in SI and the AES system is the derived unit of force.
- In SI, the derived force unit is the newton (N), based on the natural force unit of $kg \cdot m/s^2$.
- In the AES, a choice can be made to select an arbitrary unit of force or an arbitrary unit of mass.
- Newton's law automatically fixes the other unit:

$$\text{Force} = \text{mass} \cdot \text{acceleration} \qquad \text{(Eq.1)}$$

- If the pound is chosen as the mass unit (lb_m), it may be expressed in terms of the kilogram; the lb_m has 0.4536 times the mass of a kg. Then the fundamental derived unit of force is that which produces an acceleration in 1 lb_m of 1 ft/s^2. This unit is the *poundal,* with dimensions of ft/s^2.
- If the pound is chosen as the fundamental unit of force, the lb_f is the unit of force that will give a lb_m an acceleration of 32.174 ft/s^2. It is also the force of gravity between the lb_m and earth existing at sea level and 45° latitude (the standard location). When the lb_f is selected as the unit of force, the derived unit of mass is that mass which will be accelerated at the rate of 1 ft/s^2 when acted on by 1 lb_f. This derived unit of mass is known as the *slug,* with a mass of 14.5939 kg, and units of $lb_f \cdot sec^2/ft$.
- A lb_f gives a lb_m an acceleration of 32.174 ft/s^2.

- Unfortunately, engineers have selected the lb_f as the unit of force, and the lb_m as the unit of mass. When these are substituted in (Eq.1), the resulting equation is neither algebraically or dimensionally correct. To avoid this incongruity, (Eq.1) must be rewritten as:

$$\text{Force} = \text{mass} \cdot \text{acceleration}/g_c \qquad \text{(Eq.2)}$$

where g_c is a constant equal to 32.174 $lb_m \cdot ft/(lb_f \cdot sec^2)$, and is independent of location.

- The weight of a body is the force of gravity existing between the body and earth, and since weight is a force, we express it in terms of the lb_f when the AES is used. Fortunately, the variations in weight produced by latitude or elevation are small, rarely exceeding 0.25%. Thus, (Eq.2) is rewritten as:

$$F = W \cdot a/g \qquad \text{(Eq.3)}$$

Example 1. Mass vs Weight

- Calculate the mass of a block of aluminum with a volume of 0.1500 m^3 (5.297 ft^3) at the standard location. Calculate the gravitational force acting on the block, and the stress in a 0.500 cm (0.1969 in) diameter wire suspending the block. Make the calculations in SI and AES units.
- *Data.* The mass density of aluminum is 2702 kg/m^3 (168.7 lb_m/ft^3). Stress has the units of force per unit area (or pressure), expressed as Pa or lb_f/in^2. Assume the mass of the wire and air buoyancy can be neglected.

$$g = 9.8066\ m/s^2 = 32.174\ ft/s^2$$

Mass vs Weight for SI

Mass of the block is $(2702\ kg/m^3)(0.1500\ m^3) = 405.3\ kg$

Weight is

$W = m \cdot g$

$W = (405.3\ kg)(9.8066\ m/s^2) = 3975\ kg.m/s^2 = 3975\ N$

Stress has units of force per area.

Area of wire is $\pi(0.25/100)^2 = 1.964 \times 10^{-5}\ m^2$

$\sigma = F/A = 3975\ N / 1.964 \cdot 10^{-5}\ m^2 = 2.024 \times 10^8\ Pa$

Mass vs Weight for AES

Mass of the block is $(168.7\ lb_m/ft^3)(5.297\ ft^3) = 893.6\ lb_m$

!!! Weight is tricky !!!

the numerical value of the weight in lb_f = the weight in lb_m when the acceleration of gravity equals gc so:

$$W = m \cdot g$$

$$W = 168.7\ lb_m/ft^3)(5.297\ ft^3) = 893.6\ lb_m = 893.6\ lb_f$$

Stress has units of force per area.

Area of wire is $\pi(0.09845)^2 = 0.03043\ in^2$

$$\sigma = F/A = 893.6\ lb_f / 0.03043\ in^2 = 2.937 \times 10^4\ lb_f / in^2$$

Example 2. Kinetic Energy

- Calculate the kinetic energy (as J and $ft \cdot lb_f$) of 1 mole of oxygen traveling at a linear speed of 101 m/s (331.4 ft/s).
- *Data.* The mass of a mole of O_2 is 0.0320 kg (0.07055 lb_m).

The kinetic energy (E_k) is $E_k = ½\, m\, V^2$

$$E_k\ (SI) = ½\ (0.0320\ kg)(101\ m/s)^2 = 163\ J$$

$$E_k\ (AES) = ½\ (0.07055\ lb_m)(331.4\ ft/s)^2 = 3874.11\ lb_m \cdot ft^2/s^2$$

A lb_f gives a lb_m an acceleration of 32.174 ft/s^2.

$$1\ lb_f = 32.174\ lb_m \cdot ft/s^2$$

$$E_k(AES)=(3874.11\ lb_m \cdot ft^2/s^2)/(32.174\ lb_m \cdot ft/lb_f \cdot s^2)= 120\ ft \cdot lb_f$$

Example 3. Energy of Lifting

- Calculate the work in kW · h (commonly designated kWh) required to raise 1 tonne of iron ore (1000 kg) a distance of 30 m at the standard location.
- *Data.* Work is the product of force and distance. The force in this case is the acceleration of gravity times the mass. This is equivalent to the weight in newtons.

$$\text{Work} = (1000\ \text{kg})(9.8066\ \text{m/s}^2)(30\ \text{m}) = 294200\ \text{N·m} = 294200\ \text{J}$$

a joule is a watt · second

$$\text{Work} = (294\,200\ \text{W·s})/[(3600\ \text{s/hr})(1000\ \text{W/kW})] = 0.0817\ \text{kWh}$$

Conversion of Units

- Two types of conversions are required.
  - In the first, multiplication or division converts one unit into another by the conversion factor.
  - In the second, it is necessary to use the conversion factor plus addition or subtraction of an additional term.

Conversion of Units

	cm	**METER**	km	in	ft	mile
1 centimeter =	1	10^{-2}	10^{-5}	0.3937	3.281×10^{-2}	6.214×10^{-6}
1 **METER** =	100	1	10^{-3}	39.37	3.281	6.214×10^{-4}
1 kilometer =	10^{5}	1000	1	3.937×10^{-4}	3281	0.6214
1 inch =	2.540	2.540×10^{-2}	2.540×10^{-3}	1	8.333×10^{-2}	1.578×10^{-5}
1 foot =	30.48	0.3048	3.048×10^{-4}	12	1	1.894×10^{-4}
1 statute mile =	1.609×10^{5}	1609	1.609	6.336×10^{4}	5280	1

1 foot = 1200/3937 meter
1 meter = 3937/1200 ft
1 angstrom (Å) = 10^{-10} meter
1 X-unit = 10^{-13} meter
1 micron = 10^{-6} meter
1 millimicron (mu) = 10^{-9} meter

1 light-year = 9.460×10^{12} km
1 par-sec = 3.084×10^{13} km
1 fathom = 6 ft
1 yard = 3 ft
1 rod = 16.5 ft
1 mil = 10^{-3} in

1 nautical mile = 1852 meters = 1.1508 statute miles
1 nautical mile = 6076.10 ft

Conversion of Units

LENGTH

millimeter (mm)
centimeter (cm)
meter (m)
kilometer (km)

inch (in. ")
foot (ft. ')
yard (yd)
mile (mi)

10^3 mm = 10^2 cm = 1 m
10^3 m = 1 km

36 in. = 3 ft = 1 yd
5280 ft = 1760 yd = 1 mi

$$\text{mm} \xleftarrow{25.40} \text{in.} \qquad \text{mm} \xrightarrow{0.0394} \text{in.}$$

$$\text{m} \xleftarrow{0.3048} \text{ft} \qquad \text{m} \xrightarrow{3.281} \text{ft}$$

$$\text{m} \xleftarrow{0.9144} \text{yd} \qquad \text{m} \xrightarrow{1.094} \text{yd}$$

$$\text{km} \xleftarrow{1.609} \text{mi} \qquad \text{km} \xrightarrow{0.6214} \text{mi}$$

Example 4. Units of Energy

- The heat of formation of compounds is frequently listed in units of cal/mole. Use a dimension table to calculate the conversion factor for changing the heat of formation of CO_2 from cal/mol to kWh/kg.

The molar mass of CO_2 = (12.01 + 32) = 44.01 g

cal	mol	1000 g	4.184 J	W · s	h	kW
mol	44.01 g	kg	cal	J	3600 s	1000 W

The conversion factor is then 2.6408×10^{-5}.

Conversion Equations: Temperature and Pressure

- Some unit conversions cannot be accomplished by use of a dimension table. For this case, the addition or subtraction of a quantity must accompany the multiplication of conversion factors.
- Two examples of dimensions that often require conversion equations are temperature and pressure.
- Conversion equations are required here because there are units with different sizes and different zero points.

Temperature

- The SI unit of temperature is the kelvin, designated K, which is an absolute scale. Its zero point is absolute zero. The absolute scale for the AES is the Rankine scale, designated R, whose zero point is also absolute zero. The conversion factor for these two scales is:

$$K = °R/1.8$$

- Temperature scales such as Fahrenheit and Celsius are relative temperature scales because the zero points of these scales are fixed at different arbitrary standards. For Celsius, the zero point is fixed at 273.15 K, which is 0.01 K below the triple-point of water. For Fahrenheit the zero point is 255.372 K.

Temperature

$$1.0\ \Delta K = 1.0\ \Delta°C; \quad 1.0\ \Delta°R = 1.0\ \Delta°F$$

$$1.0\ \Delta°C = 1.8\ \Delta°F; \quad 1.0\ \Delta K = 1.8\ \Delta°R$$

$$T(K) = t(°C) + 273.15$$

$$T(°R) = t(°F) + 459.67$$

$$t(°F) = 1.8t(°C) + 32$$

Pressure

- Pressure is defined as force per unit area (in the SI system, N/m^2). The SI unit for pressure is the pascal (Pa). A pascal is a very small unit of pressure. To get an idea of the intensity of a Pa, the force created by a column of water 1 mm high is 10 Pa. Another reference point is that 1 atm is approximately 100 kPa, and 1 bar = 100 kPa
- Pressure is similar to temperature in that it can be expressed in absolute or relative scales, with several different units. Absolute scales base pressure readings on a perfect vacuum or a completely evacuated reference point for zero pressure. Relative scales have the same units, but with the zero point being 1 standard atm (1.013×10^5 Pa).

Pressure

	atm	dyne/cm^2	inch of water	cm Hg	NT/METER^2	lb/in^2	lb/ft^2
1 atmosphere =	1	1.013×10^6	406.8	76	1.013×10^5	14.70	2116
1 dyne per cm^2 =	9.869×10^{-7}	1	4.015×10^{-4}	7.501×10^{-5}	0.100	1.450×10^{-5}	2.089×10^{-3}
1 inch of water at 4°C[a] =	2.458×10^{-3}	2.491	1	0.1868	249.1	3.613×10^{-2}	5.202
1 centimeter of mercury at 0°C[a] =	1.316×10^{-2}	1.333×10^4	5.353	1	1333	0.1934	27.85
1 NEWTON per METER^2 =	9.869×10^{-6}	10	4.015×10^{-3}	7.501×10^{-4}	1	1.450×10^{-4}	2.089×10^{-2}
1 pound per in^2 =	6.805×10^{-2}	6.895×10^4	27.68	5.171	6.895×10^3	1	144
1 pound per ft^2 =	4.725×10^{-4}	478.8	0.1922	3.591×10^{-2}	47.88	6.944×10^{-3}	1

Other Conversion tables

UNITS AND CONVERSION FACTORS

E.J. ROSCHKE
PROPULSION DIVISION
JET PROPULSION LABORATORY

1