Assume that the temperature attained is at least 941 K. The heat required to raise the temperature of 1 mole of $U_{(\alpha)}$ and 2 moles of MgF_2 from 298 to 941 is

$$\Delta H_{(i)} = \int_{298}^{941} \left(c_{p,U(\alpha)} + 2c_{p,MgF_2(s)}\right) dT$$
$$= [(25.10 + 2 \times 77.11)(941 - 298)]$$
$$+ \left[(2.38 + 2 \times 3.89)\frac{10^{-3}}{2}(941^2 - 298^2)\right]$$
$$+ 2 \times 14.94 \times 10^5 \left(\frac{1}{941} - \frac{1}{298}\right)$$
$$= 118{,}866 \text{ J}$$

which is less than the 328,800 J released by the exothermic reaction. Assume, now, that the temperature attained is at least 1049 K. The heat required to transform 1 mole of U from α to β at 941 K and heat 1 mole of $U_{(\beta)}$ and 2 moles of MgF_2 from 941 to 1049 K is

$$\Delta H_{(ii)} = \Delta H_{U,(\alpha \to \beta)} + \int_{941}^{1049} (c_{p,U(\beta)} + 2c_{p,MgF_2})dT$$
$$+ 2800 + [(42.93 + 2 \times 77.11)(1049 - 491)]$$
$$+ \left[\frac{2 \times 3.98 \times 10^{-3}}{2}(1049^2 - 491^2)\right]$$
$$+ 2 \times 14.94 \times 10^5 \left(\frac{1}{1049} - \frac{1}{491}\right)$$
$$= 24{,}601 \text{ J}$$

Heating to 1049 K consumes 118,866 + 24,601 = 143,467 J, which leaves 328,800 − 143,467 = 185,333 J of sensible heat available for further heating. Assume that the temperature attained is at least 1408 K. The heat required to transform 1 mole of U from β to γ at 1049 K and increase the temperature of 1 mode of $U_{(\gamma)}$ and 2 moles of MgF_2 from 1049 K is

$$\Delta H_{(iii)} = \Delta H_{U,(\beta \to \gamma)} + \int_{1049}^{1408} (c_{p,U(\gamma)} + 2c_{p,MgF_2})dT$$
$$= 4800 + [(32.28 + 2 \times 77.11)(1408 - 1049)]$$
$$+ \left[\frac{2 \times 3.98 \times 10^{-3}}{2}(1408^2 - 1049^2)\right]$$
$$+ 2 \times 14.94 \times 10^5 \left(\frac{1}{1408} - \frac{1}{1049}\right)$$
$$= 76{,}612 \text{ J}$$

To reach 1408 K, requires 143,467 + 76,612 = 151,079 of the available heat, which leaves 108,721 J for further heating. Assume that the temperature reaches 1536 K.

The heat required to melt 1 mole of U at 1408 K and increase the temperature of 1 mole of liquid U and 2 moles of MgF_2 from 1408 to 1536 K is

$$\Delta H_{(iv)} = \Delta H_{U,(m)} + \int_{1408}^{1536} (c_{p,U(l)} + 2c_{p,MgF_2})dT$$
$$= 9200 + [(48.66 + 2 \times 77.11)(1536 - 1408)]$$
$$+ \left[\frac{2 \times 3.98 \times 10^{-3}}{2}(1536^2 - 1408^2)\right]$$
$$+ (2 \times 14.94 \times 10^5)\left(\frac{1}{1536} - \frac{1}{1408}\right)$$
$$= 36{,}457 \text{ J}$$

To reach 1536 K thus requires 151,079 + 36,457 = 187,536 J, which leaves 108,271 − 36,457 = 72,264 J. The remaining sensible heat is less than the molar heat of melting of MgF_2, (ΔH_m = 58,600 J), and thus is used to melt

$$\frac{72{,}264}{58{,}600} = 1.23$$

moles of MgF_2 at its melting temperature of 1536 K. The reaction products are thus liquid U, liquid MgF_2, and solid MgF_2 occurring in the ratio 1:1.23:0.77 at 1536 K.

The attainment, by the reaction products, of a final temperature of 1773 K requires that an extra 101,291 J be supplied to the adiabatic reaction container, and this is achieved by preheating the reactants to some temperature before allowing the reaction to occur. The required temperature, T, is obtained from

$$101{,}291 = \int_{298}^{T} c_{p,\text{reactants}}\, dT$$

Assume that T is less than the melting temperature of Mg, $T_{Mg(m)}$ = 923 K. The required thermochemical data are

$$c_{p,Mg} = 21.12 + 11.92 \times 10^{-3}\,T + 0.15 \times 10^5\,T^{-2} \quad \text{in the range 298–923 K}$$
$$c_{p,UF_4} = 107.53 + 29.29 \times 10^{-3}\,T - 0.25 \times 10^5\,T^{-2} \quad \text{in the range 298–1118 K}$$

Thus

$$101{,}291 = \int_{298}^{T} (2c_{p,Mg} + c_{p,UF_4})dT$$
$$= [(2 \times 21.13 + 107.53)(T - 298)]$$
$$+ \left[(2 \times 11.92 + 29.29)\frac{10^{-3}}{2}(T^2 - 298^2)\right]$$
$$- (2 \times 0.15 - 0.25) \times 10^5 \left(\frac{1}{T} - \frac{1}{298}\right)$$