

2015-2016 Spring Semester Material and Energy Balance

Fundamentals of Material Balances with Applications to Non-Reacting Systems

Assist. Prof. Dr. Murat Alkan

08.03.2016

4th Week

Introduction

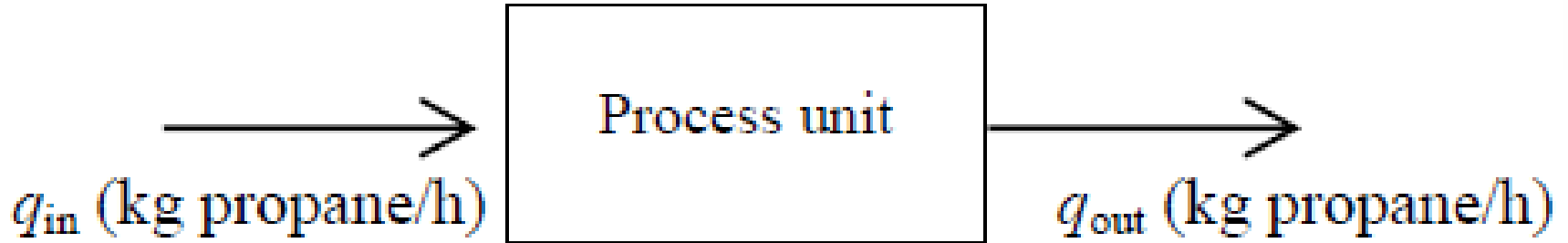
- Material balances are important first step when **designing** a new process or **analyzing** an existing one.
- Material balances are nothing more than the application of the law of conservation of mass, which states that mass can **neither** be created **nor** destroyed.

total mass of input = total mass of output.

- A material balance is an accounting for material.
- Thus, material balances are often compared to the balancing of current accounts.
- They are used in industry to calculate mass flow rates of different streams entering or leaving chemical or physical processes.

The General Balance Equation

- Suppose propane is a component of both the input and output streams of a continuous process unit shown below, these flow rates of the input and output are measured and found to be different.



- If there are no leaks and the measurements are correct, then the other possibilities that can account for this difference are that propane is either being generated, consumed, or accumulated within the unit.

The General Balance Equation

- A balance on a material in a system (a single process unit, a collection of units, or an entire process) may be written in the following general way:

$$\begin{array}{ccccccccc} \textit{Input} & + & \textit{generation} & - & \textit{output} & - & \textit{consumption} & = & \textit{accumulation} \\ \text{(enters} & & \text{(produced} & & \text{(leaves} & & \text{(consumed} & & \text{(buildup} \\ \text{through} & & \text{within} & & \text{through} & & \text{within} & & \text{within} \\ \text{system} & & \text{system} & & \text{system} & & \text{system)} & & \text{system)} \\ \text{boundaries)} & & \text{boundaries)} & & \text{boundaries)} & & & & \end{array}$$

- This general balance equation may be written for any material that enters or leaves any process system;
 - it can be applied to the total mass of this material or to any molecular or atomic species involved in the process.

The General Balance Equation

$$\text{Accumulation} = \text{Input} + \text{Generation} - \text{Output} - \text{Consumption}$$

- **Input**: All flows into the system.
- **Generation**: Material produced by chemical reaction in the system.
- **Output**: All flows leaving the system.
- **Consumption**: Material consumed by chemical reaction in the system.
- **Accumulation**: The change of material in the system with time.
- Accumulation is positive for material increasing or building up with time and negative for material being depleted over time.

The General Balance Equation

$$\text{Accumulation} = \text{Input} + \text{Generation} - \text{Output} - \text{Consumption}$$

At steady state;

$$\text{Accumulation} = 0$$

$$\text{Input} + \text{Generation} = \text{Output} + \text{Consumption}$$

If there is no chemical reaction;

$$\text{Generation} = 0$$

$$\text{Consumption} = 0$$

$$\text{Accumulation} = \text{Input} - \text{Output}$$

If there is no flow across system boundaries;

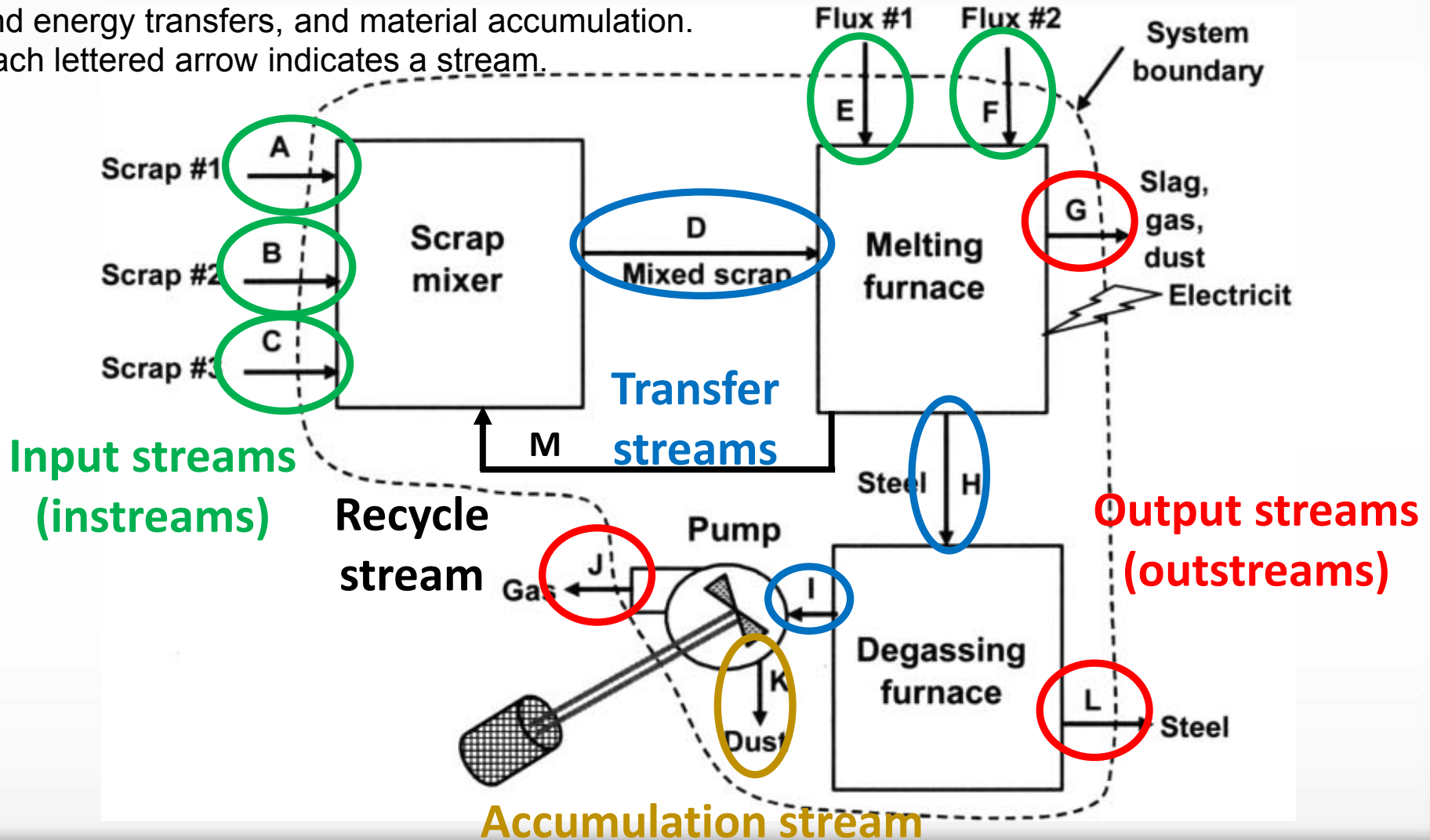
$$\text{Accumulation} = \text{Generation} - \text{Consumption}$$

Process Classifications

- Processes can be classified as batch, semi-batch, continuous, steady-state or transient.
- Batch processes. Raw materials are added to a vessel at the beginning of the process, and kept there until the desired final state is reached.
 - An example is the metallothermic reduction of TiCl_4 by Mg.
- Semi-batch processes. Some reactants are put into a vessel, and then other substances are added steadily until the desired final state is reached.
 - One example is the passage of a drying gas over a batch of wet sludge.
- Continuous process. The in- and outstream material flows continuously throughout the duration of the process.
 - Production of Portland cement in a rotary kiln, the electroplating of zinc on steel, and the production of glass.

Streams

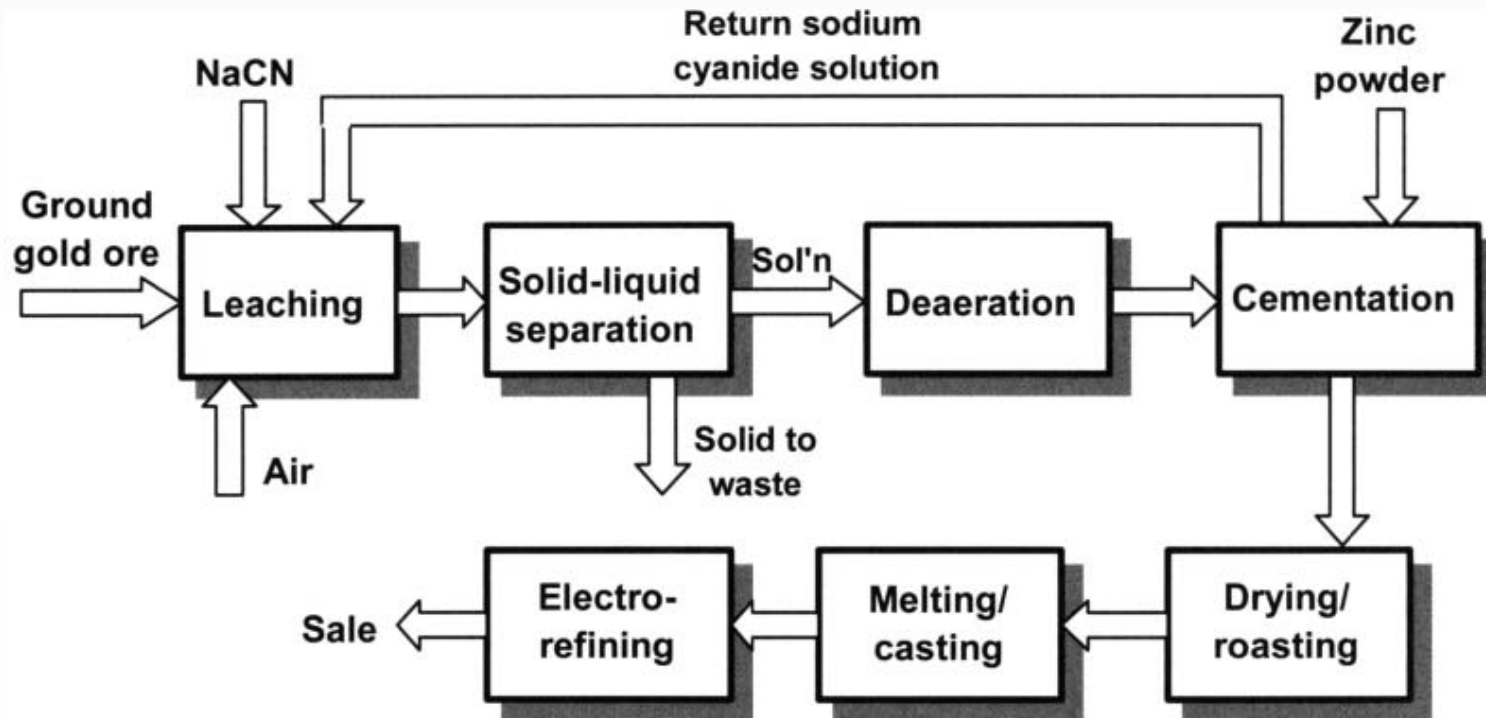
Schematic diagram of a system showing material and energy transfers, and material accumulation. Each lettered arrow indicates a stream.



Flowsheets

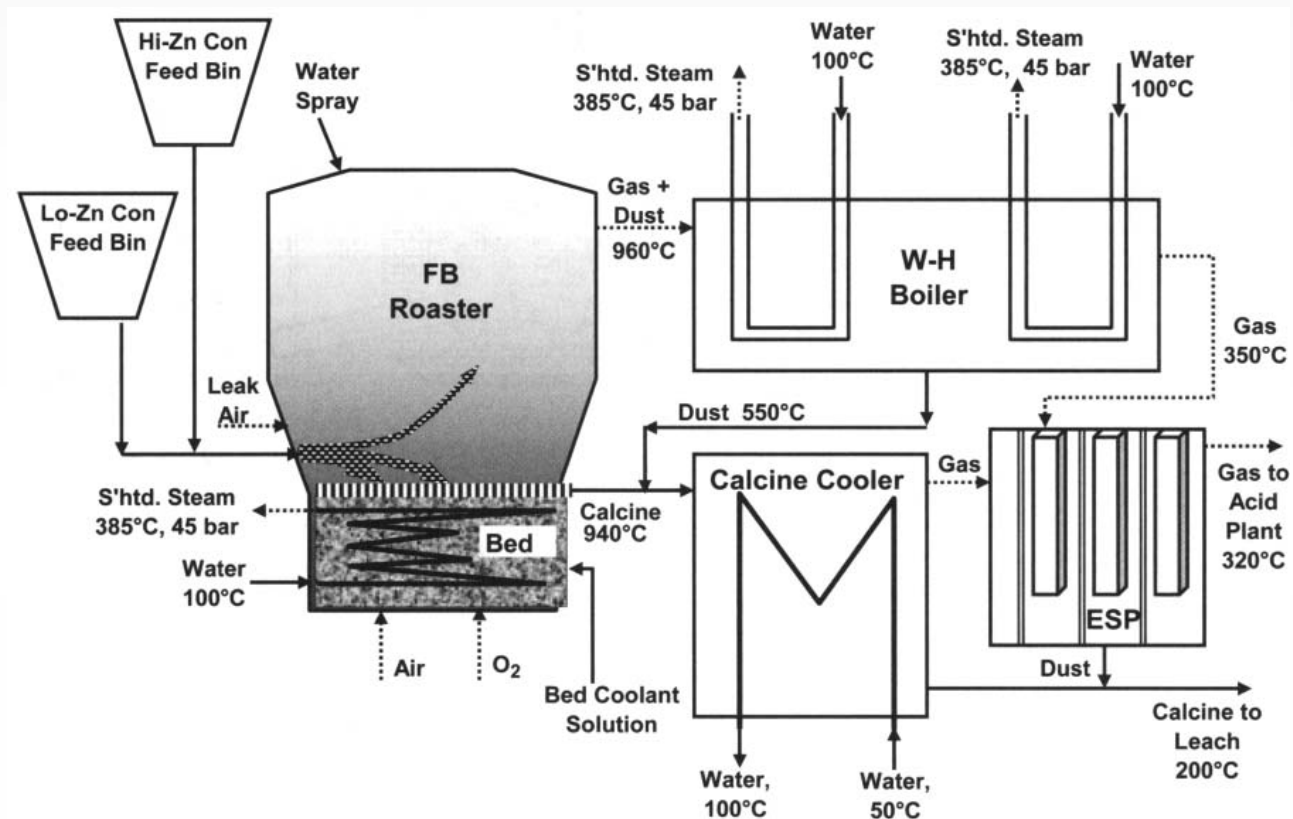
- A written description of a process can contain all of the essential information needed for making a material balance.
- However, a textual description is not a good way to present information that contains a lot of numerical data and technical information.
- A better way is to present the same information diagrammatically.
- Such a diagram is called a flowsheet, and consists of boxes or other symbols to represent process devices, and lines with arrows to represent streams.

Flowsheet Examples



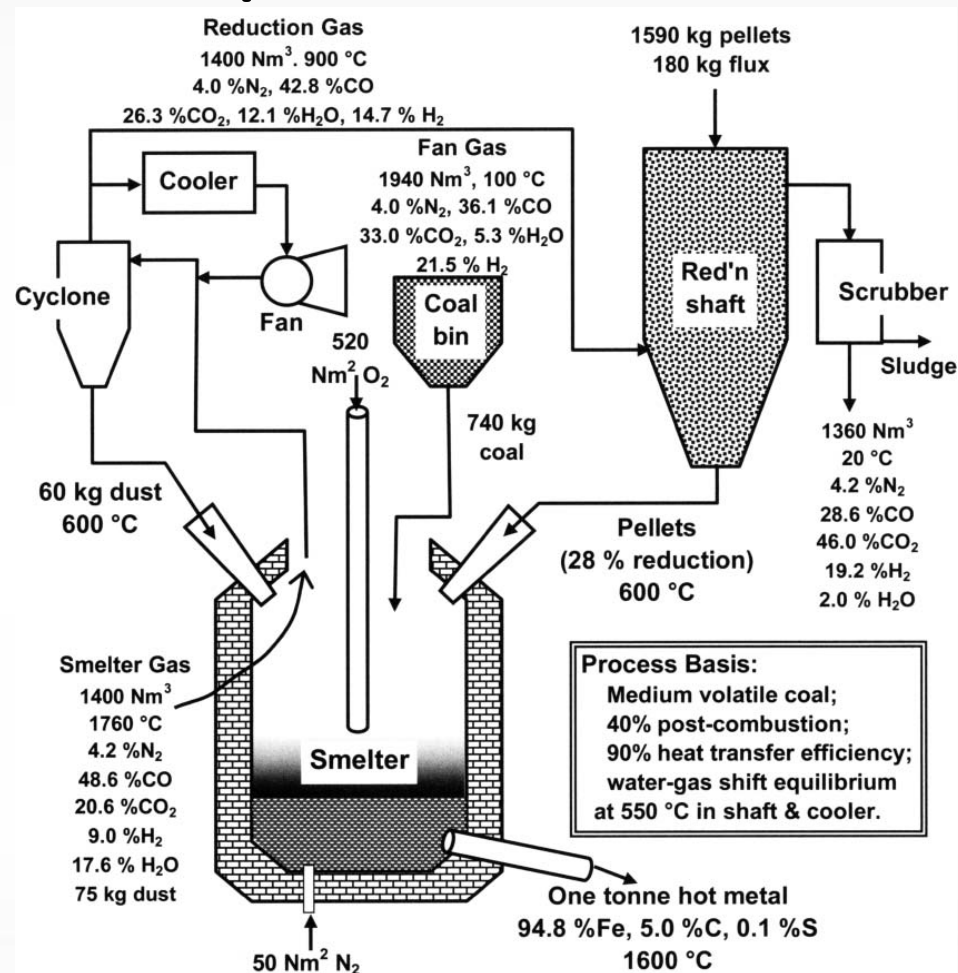
Typical indicative flowsheet for the production of gold from ore

Flowsheet Examples



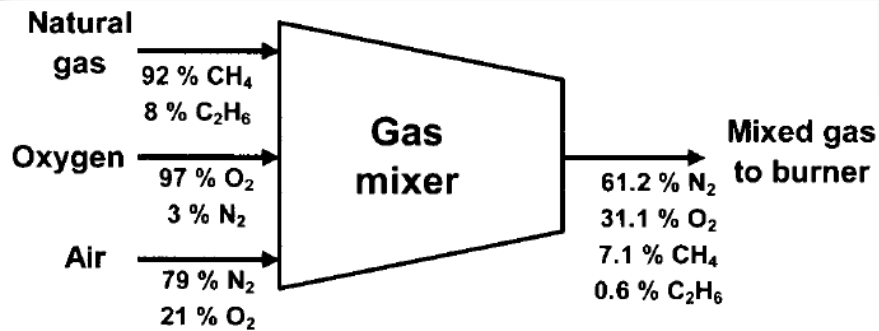
Flowsheet for fluid bed roasting of zinc concentrate using shapes to describe the various devices. Dashed lines indicate the flow of gas, and solid lines the flow of solids. Calcine is the oxide product of roasting. A waste-heat boiler (designated W-H) recovers heat from the gas to generate steam. The ESP is an electrostatic precipitator for final dust removal.

Flowsheet Examples

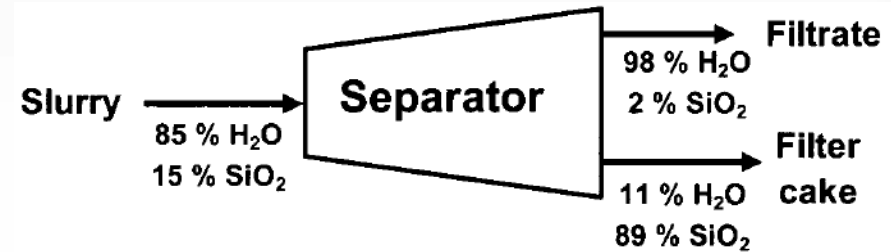


Simplified flowsheet for the AISI-DOE process for direct smelting of iron ore.

Flowsheet Examples



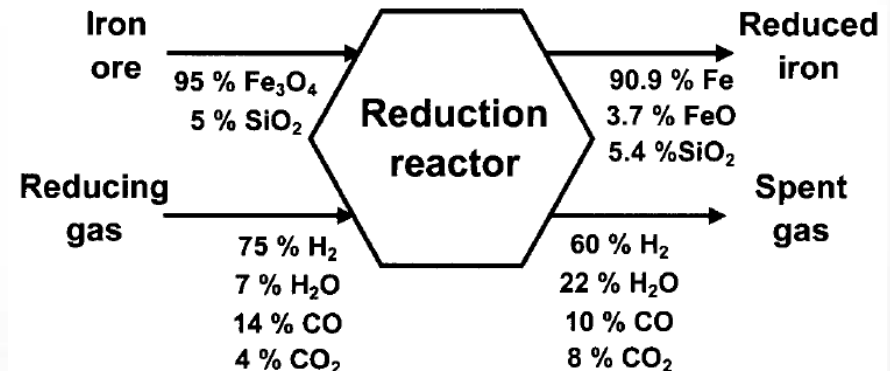
Flowsheet showing the use of a mixer device to prepare combustion gas for burner.



Flowsheet showing the use of a separator device (here, a drum filter) to separate solid and liquid phases



Flowsheet showing use of a separator device to remove moisture from air.



Flowsheet showing the use of a chemical reactor device to reduce iron oxide.

Procedure for Material Balance Calculations

- In material balance problems, you will usually be given a description of a process, the values of several process variables, and a list of quantities to be determined.

1. **Draw and label** the process flow chart (block diagram).

When labeling, write the values of known streams and assign symbols to unknown stream variables. Use the minimum number possible of symbols.

2. **Select a basis of calculation.**

This is usually the given stream amounts or flow rates, if no given then assume an amount of a stream with known composition.

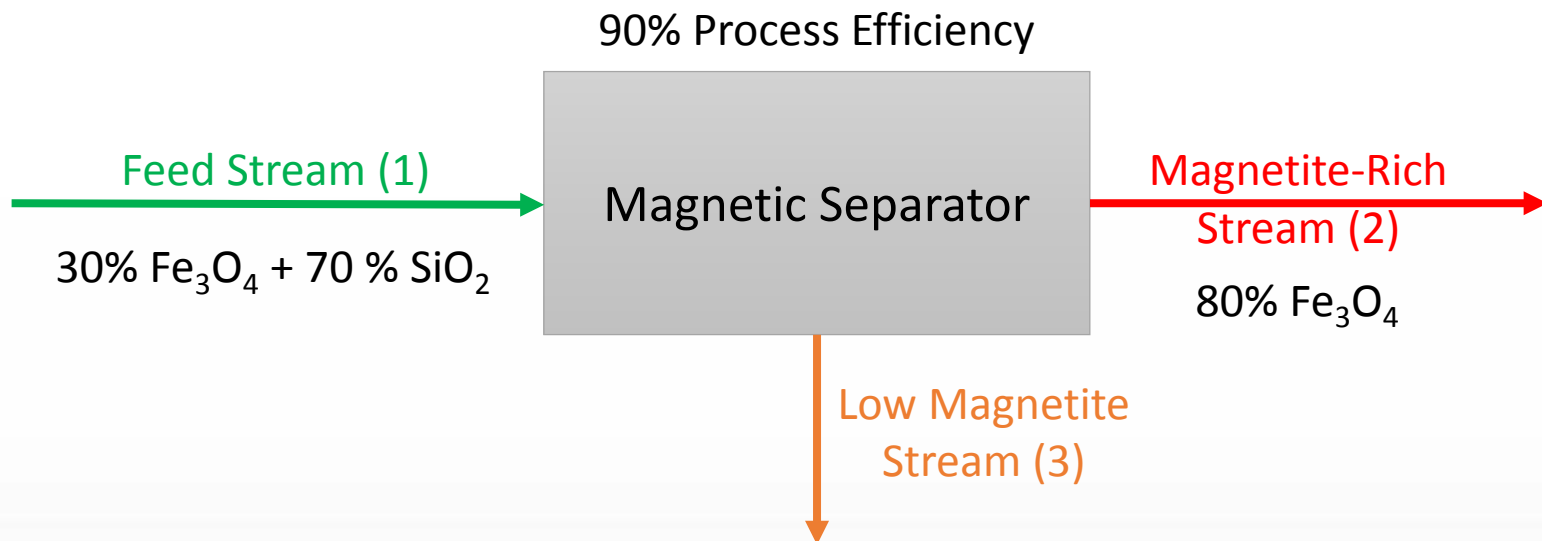
3. Write **material balance equations.**

Note in here the maximum number of independent equations you can write for each system is equal the number of species in the input and output streams of this system.

4. **Solve the equations** derived in step 3 for the unknown quantities to be determined..

Example 1

- An ore containing Fe_3O_4 (30wt.%) and SiO_2 is separated by a magnetic separator into two streams; one rich in magnetite (80wt.%) and one depleted in Fe_3O_4 . If the separation efficiency is 90%, calculate the material balance for this separating process where the ore has 1000 kg/h mass flow rate.



$$\text{Accumulation} = \text{Input} + \text{Generation} - \text{Output} - \text{Consumption}$$

Example 1

$$\text{Accumulation} = \text{Input} + \text{Generation} - \text{Output} - \text{Consumption}$$

$$\text{Accumulation} = 0$$

$$\text{Generation} = 0$$

$$\text{Consumption} = 0$$

$$\text{Input} = \text{Output}$$

| Information | Stream | | |
|-----------------------------------------------|--------------|--------------|------------|
| | (1) | (2) | (3) |
| Fe ₃ O ₄ mass flow rate | 30% 300 kg/h | 80% 270 kg/h | 30 kg/h |
| SiO ₂ mass flow rate | 70% 700 kg/h | 67.5 kg/h | 662.5 kg/h |
| Total Mass Flow Rate | 1000 kg/h | S2 | S3 |

90% efficiency

$$300 \times 90/100 = 270 \text{ kg/h Fe}_3\text{O}_4$$

$$300 - 270 = 30 \text{ kg/h Fe}_3\text{O}_4$$

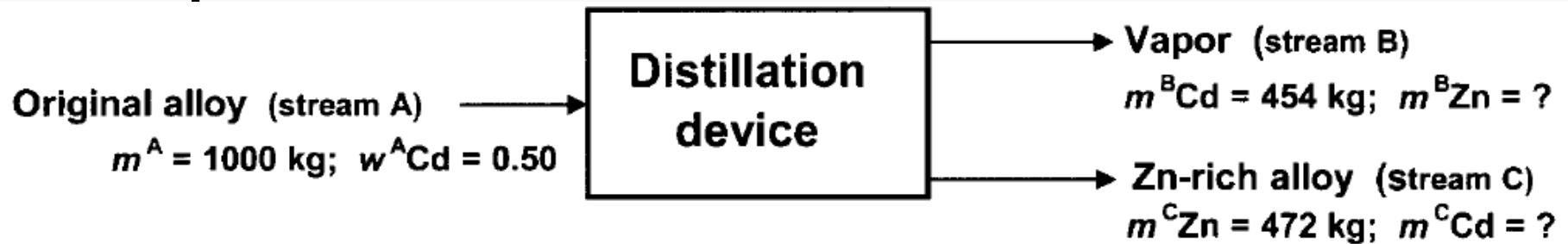
$$270 \times 20/80 = 67.5 \text{ kg/h SiO}_2$$

$$700 - 67.5 = 662.5 \text{ kg/h SiO}_2$$

$$\text{S2} = 270 + 67.5 = 337.5 \text{ kg/h}$$

$$\text{S3} = 30 + 662.5 = 692.5 \text{ kg/h}$$

Example 2



- Figure showed that the difference in the vapor pressures of cadmium and zinc might allow the two metals to be separated by distillation. We now apply the law of conservation of mass to explore such a process.
- An alloy with a mass fraction of 50 % cadmium ($w\text{Cd} = 0.50$) enters a distillation device (a separator) at 1000 kg/h and exits as two streams.
- At steady state, the gas stream contains 454 kg/h of Cd, and the liquid stream contains 472 kg/h of Zn.
- Write elemental balances on each constituent and prepare a complete mass balance.

Example 2

Cadmium balance:

$$500 \text{ kg} = 454 \text{ kg} + m^{\text{C}}\text{Cd}$$

$$m^{\text{C}}\text{Cd} = 500 - 454 = 46 \text{ kg}$$

Zinc balance:

$$500 \text{ kg} = 472 \text{ kg} + m^{\text{B}}\text{Zn}$$

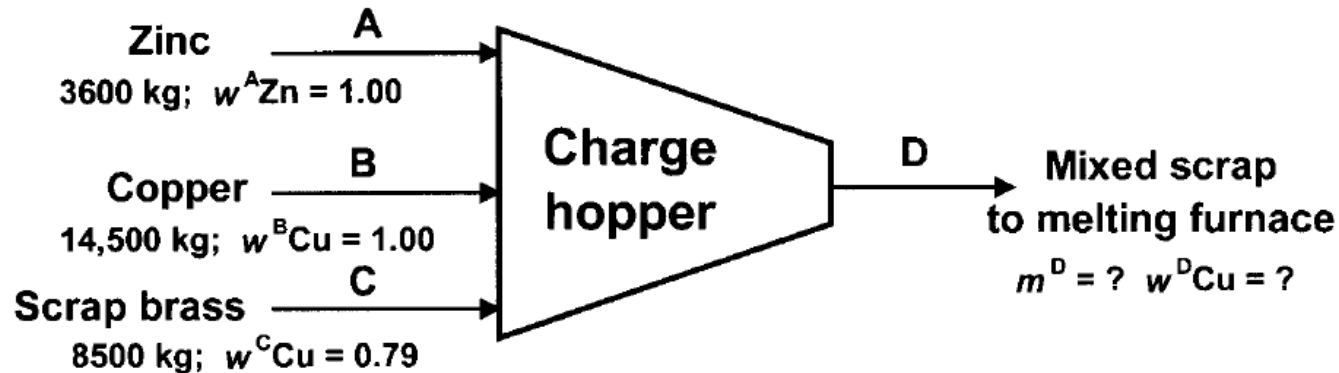
$$m^{\text{B}}\text{Zn} = 500 - 472 = 28 \text{ kg}$$

Total mass balance:

$$1000 = 454 + 46 + 472 + 28 = 1000.$$

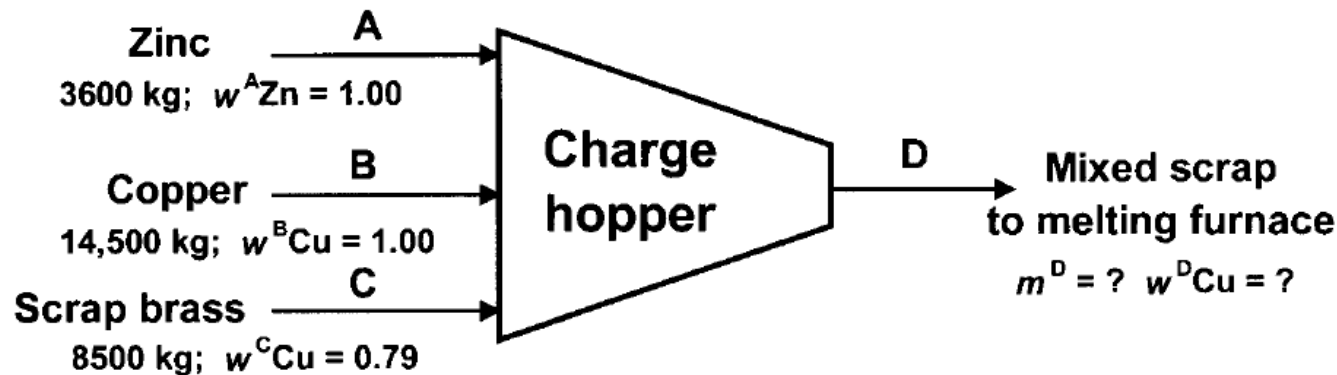
| | Stream | | |
|-----------------|--------|--------|--------|
| | A | B | C |
| <hr/> | | | |
| mass, kg | | | |
| Zn | 500 | 28 | 472 |
| Cd | 500 | 454 | 46 |
| total | 1000 | 482 | 518 |
| <hr/> | | | |
| amount, kmol | | | |
| Zn | 7.65 | 0.43 | 7.22 |
| Cd | 4.45 | 4.04 | 0.41 |
| total | 12.10 | 4.47 | 7.63 |
| <hr/> | | | |
| mass fraction | | | |
| Zn | 50% | 5.8% | 91.1% |
| Cd | 50% | 94.2% | 8.9% |
| total | 100% | 100.0% | 100.0% |
| <hr/> | | | |
| amount fraction | | | |
| Zn | 63.2% | 9.6% | 94.6% |
| Cd | 36.8% | 90.4% | 5.4% |
| total | 100.0% | 100.0% | 100.0% |

Example 3



- The preparation of an alloy for casting is done by feeding controlled amounts of raw materials to a charge hopper until it has the correct quantity for the melting furnace.
- The charge is put into a furnace, and energy is added until the alloy is melted and the final temperature is reached.
- Calculate the mass of mixed scrap in the hopper and its overall composition

Example 3

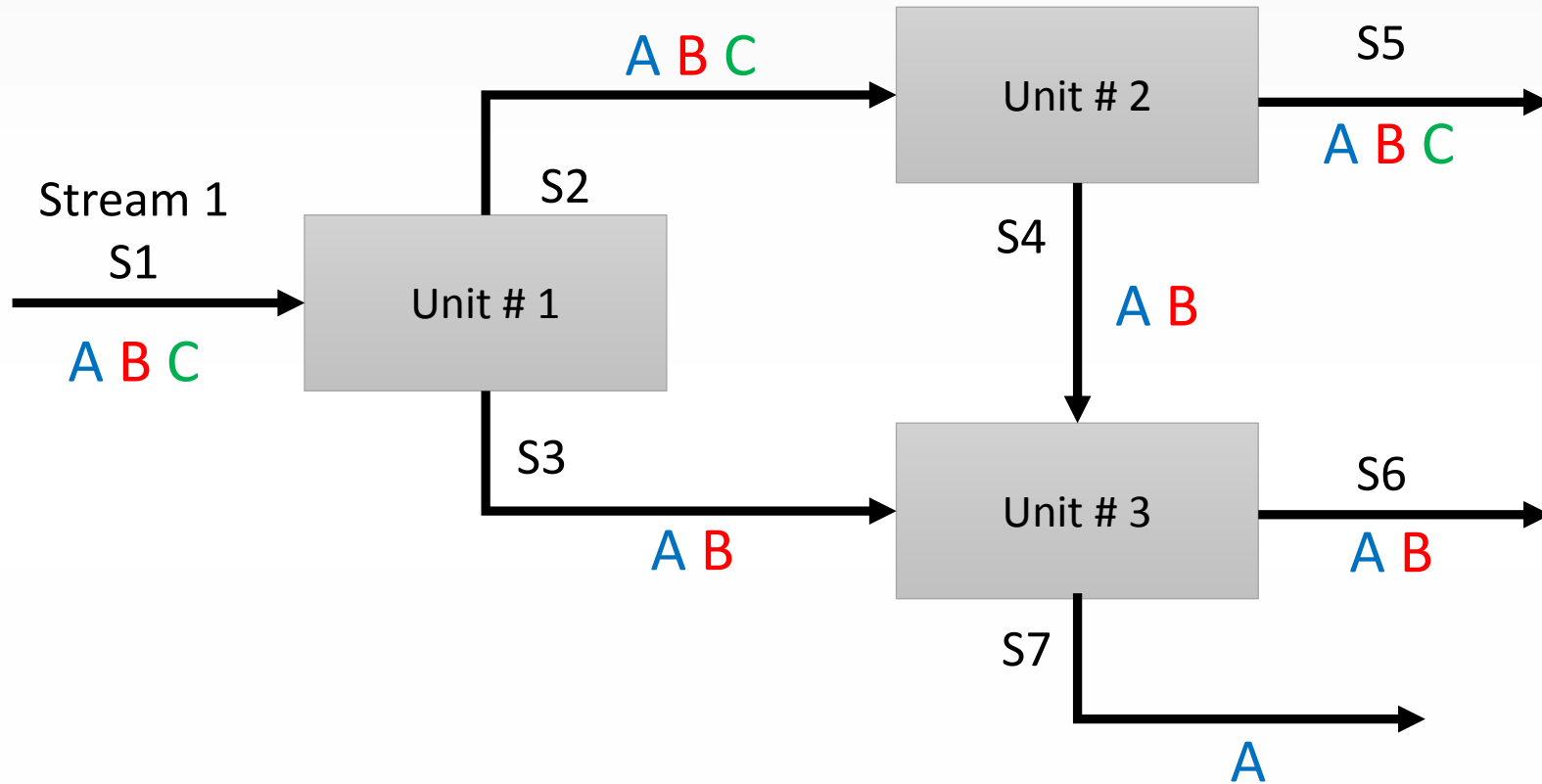


Total mass balance: $3600 + 14\,500 + 8500 = m^D = \text{kg mixture produced}$
 $m^D = 26\,600 \text{ kg}$

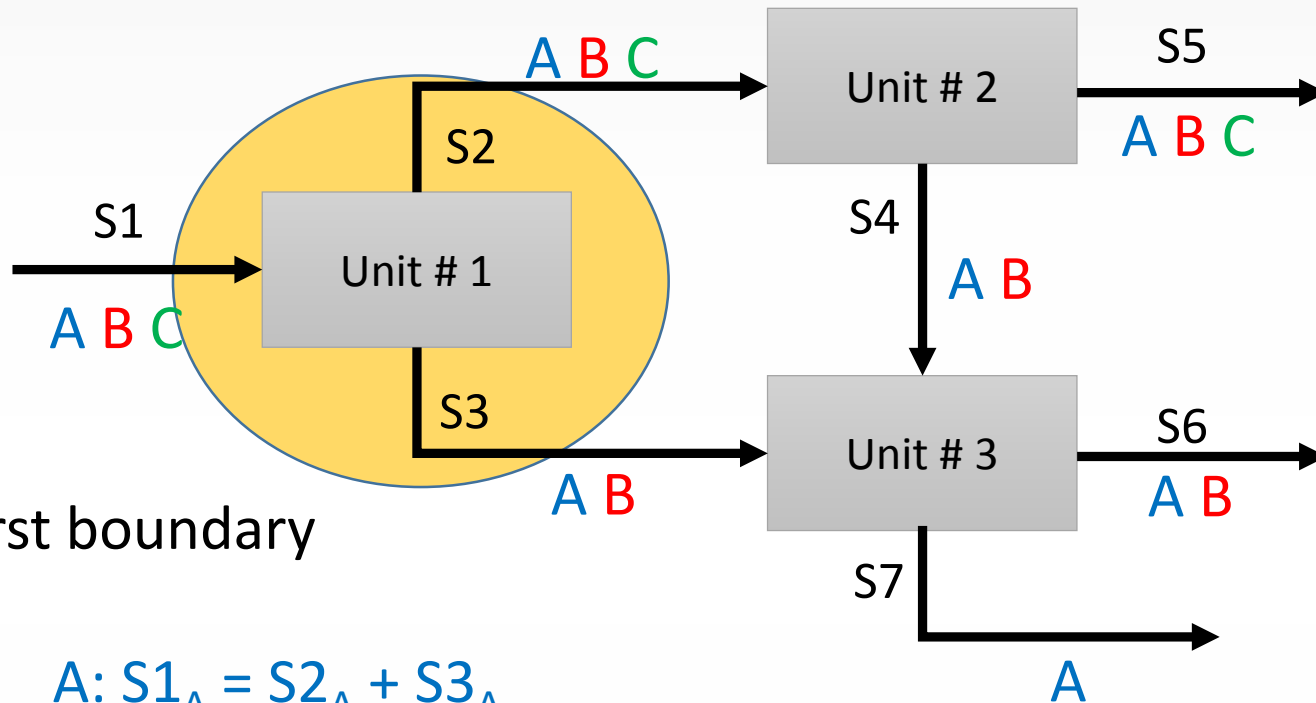
Copper balance: $14\,500 + 0.79(8500) = (26\,600)(w^D_{\text{Cu}})$
 $w^D_{\text{Cu}} = 0.798 = 79.8 \%$

Zinc balance: $3600 + 0.21(8500) = (26\,600)(1 - w^D_{\text{Cu}}) = (26\,600)(w^D_{\text{Zn}})$
 $w^D_{\text{Zn}} = 0.202 = 20.2 \%. \quad 1.00 - w^D_{\text{Zn}} = w^D_{\text{Cu}} = 0.798. \quad \underline{\text{Check}} \checkmark$

Material Balance on Multiple Unit Systems



We have 3 process units boundaries and 1 total process boundary



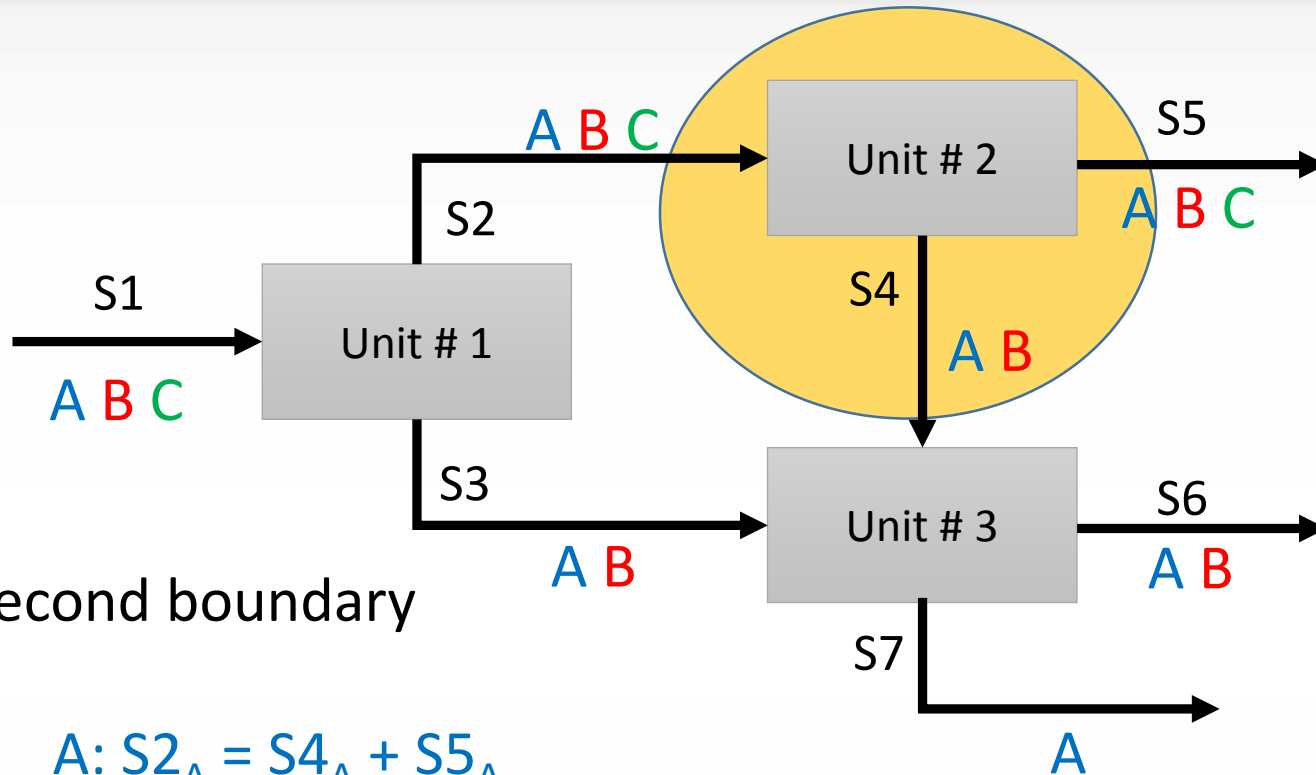
For the first boundary

$$A: S1_A = S2_A + S3_A$$

$$B: S1_B = S2_B + S3_B$$

$$C: S1_C = S2_C$$

$$\text{Total: } S1_{\text{Total}} = S2_{\text{Total}} + S3_{\text{Total}}$$



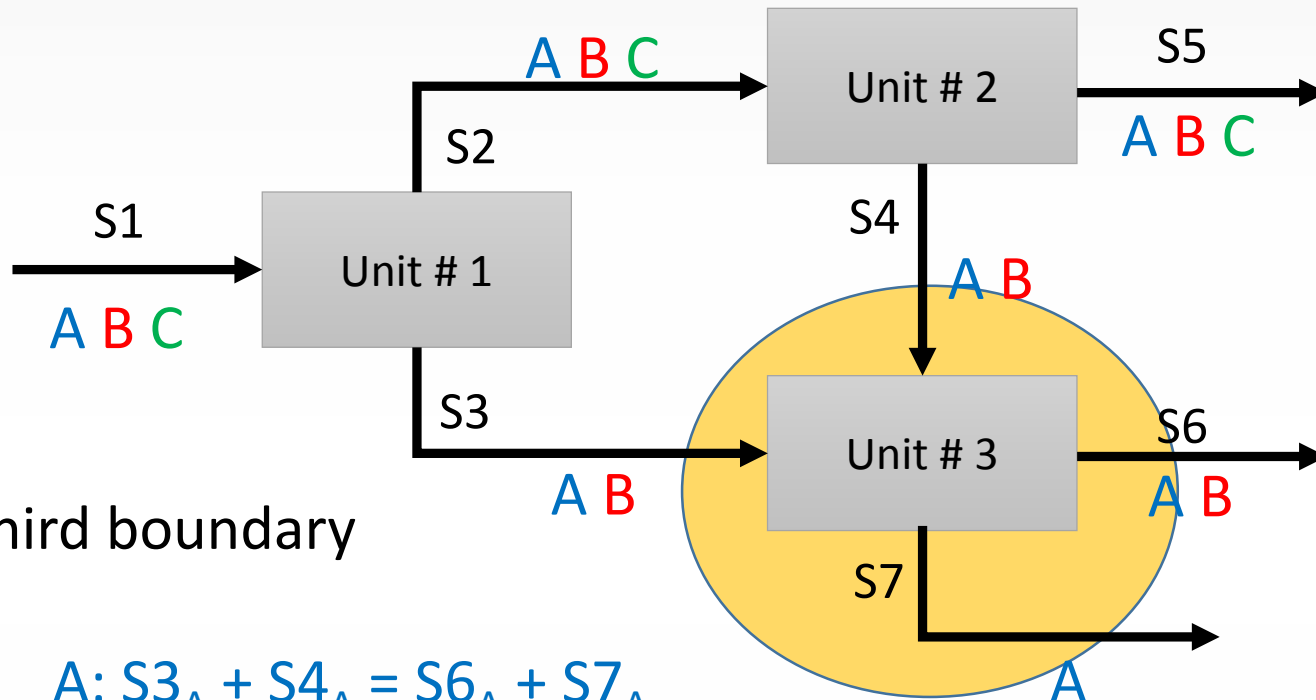
For the second boundary

$$A: S2_A = S4_A + S5_A$$

$$B: S2_B = S4_B + S5_B$$

$$C: S2_C = S5_C$$

$$\text{Total: } S2_{\text{Total}} = S4_{\text{Total}} + S5_{\text{Total}}$$

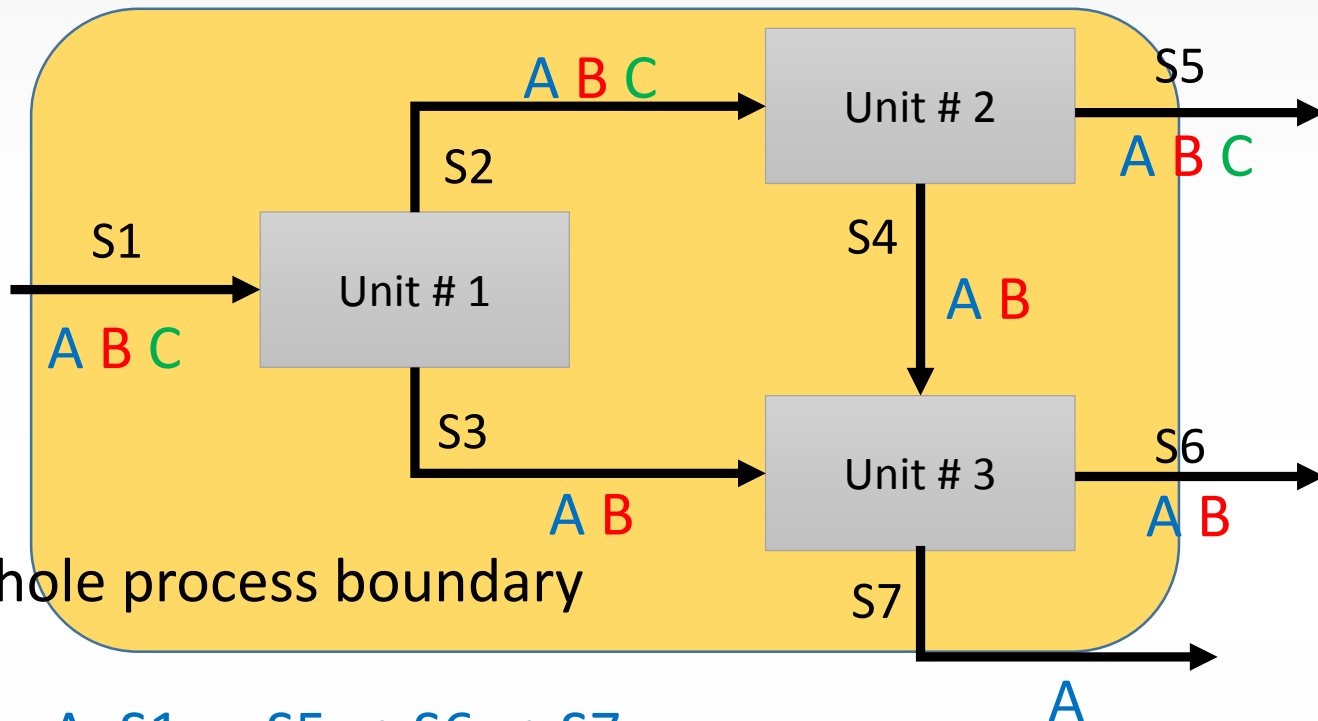


For the third boundary

$$A: S3_A + S4_A = S6_A + S7_A$$

$$B: S3_B + S4_B = S6_B$$

$$\text{Total: } S3_{\text{Total}} + S4_{\text{Total}} = S6_{\text{Total}} + S7_{\text{Total}}$$



For the whole process boundary

$$A: S1_A = S5_A + S6_A + S7_A$$

$$B: S1_B = S5_B + S6_B$$

$$C: S1_C = S5_C$$

$$\text{Total: } S1_{\text{Total}} = S5_{\text{Total}} + S6_{\text{Total}} + S7_{\text{Total}}$$

Guidelines for Setting up a Materials Balance

1. **Draw a flowsheet.** (convert text-form to diagram)
2. **Decide what kind of process is taking place.** (steady-state, batch)
3. **Set up a ledger.** (write-down the given info)
4. **Look up or obtain any necessary information that's not included in the problem statement.** (molecular mass, conversion factors)
5. **Adopt a set of consistent units for the calculations.** (convert all units to mass or amount)
6. **Choose a basis.** (if no amount given choose 100kg (or moles))
7. **Decide how many equations should be written to balance the process.**
8. **Check which variable values you can determine "by inspection".** (examine simple relations that given, and enter the results)

Guidelines for Resolving a Set of Equations

1. **Write the equations in an efficient order.** (first write equations only one unknown then solve others)
2. **Solve the equations.** (try to make one-variable eq. If the eq. is complex try using Excel)
3. **Check your answer.** (Use one of the dependent equations to check if the answer is correct)
4. **Scale the answer.** (If you used a different basis than set out in the problem statement, scale your result to the original basis)
5. **Present your results.** (Prepare a table or chart showing the results)
6. **Interpret your results.** (Summarize the findings in a useful way, and point out any unexpected or noteworthy result)

Next Week

- **Stoichiometry and the Chemical Equation**
- **Examples**